

التحريك التفاضلي، اختيار الكلي
 مادة مقاومة المواد 02
 الكلاس 1 - هندسة ميكانيكية

⑥ التمرين 01

$$U = \int_0^l \frac{M^2}{2EI} dx \quad \text{--- ①}$$

U طاقة U = 1, 2, 3

نظام في حالة التوازن

$$w = \frac{\partial U}{\partial P} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial P} dx$$

$$\frac{\partial M}{\partial P} = -x \leftarrow M = -Px \quad \text{--- ②}$$

$$w = \frac{1}{EI} \int_0^l (-Px)(-x) dx = \frac{P}{EI} \int_0^l x^2 dx$$

$$= \frac{P}{EI} \left[\frac{x^3}{3} \right]_0^l = \frac{Pl^3}{3EI}$$

نوع ③، ④

$$U = \int_0^l \frac{(-Px)^2}{2EI} dx$$

$$U = \frac{-P^2 x^3}{6EI} \Big|_0^l = \frac{-P^2 l^3}{6EI} \quad | \quad I = \frac{\pi d^4}{64}$$

$$U = -\frac{64P^2 l^3}{6E\pi d^4} \quad \text{--- ③}$$

$$U = 15,56 \times 10^3 \text{ N.m} \quad \text{--- ③}$$

الشيخ الفاضل
مكة المكرمة
الحمد لله

⑥ التحريك

$$U = \int_0^l \frac{M^2}{2EI} dx \quad \text{--- (1)}$$

الإمامة في حق نبي الله صلى الله عليه وآله وسلم

$$w = \frac{\partial U}{\partial P} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial P} dx$$

$$U = \frac{1}{EI} \int_0^L (-Pn)(-n) dn = \frac{P}{EI} \int_0^L n^2 dn$$

$$= \frac{P}{EI} \left[\frac{n^3}{3} \right]_0^L = \frac{PL^3}{3EI}$$

وہاں، وہاں

$$U = \int_0^l \frac{(-P_n)^2}{2EI} dx$$

$$U = \frac{-P^2 n^3}{6EI} = \frac{-P^2 \rho^3}{6EI} \quad | \quad I = \frac{\pi d^4}{64}$$

$$U = - \frac{64 \rho^2 l^3}{6 E \pi d^4}$$

$U = 155 \text{ kN/mm}^2$

6) التمارين

(u, v, w) : دالة على x, y, z

$$\sigma = \begin{bmatrix} 0 & 0 & \sigma_{xz} \\ 0 & 0 & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & 0 \end{bmatrix}$$

$$\sigma_{xz} = -G \sigma_y ; \sigma_{yz} = G \sigma_x ;$$

$$\sigma_x = \sigma_y = \sigma_z = \sigma_{xy} = 0$$

أي $\sigma_{xy} = G \sigma_{xy} ; \sigma_{xz} = G \sigma_{xz}$

$$\sigma_{yz} = G \sigma_{yz}$$

$$\begin{cases} \sigma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \sigma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \sigma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \end{cases}$$

$$\begin{aligned} \epsilon_x = \frac{\partial u}{\partial x} = 0 &\Rightarrow u = F_1(y, z) \quad \text{--- 1} \\ \epsilon_y = \frac{\partial u}{\partial y} = 0 &\Rightarrow v = F_2(x, z) \quad \text{--- 2} \\ \epsilon_z = \frac{\partial u}{\partial z} = 0 &\Rightarrow w = F_3(x, y) \quad \text{--- 3} \end{aligned}$$

$$\begin{aligned} \epsilon_z = \frac{\partial u}{\partial z} = 0 &\Rightarrow \frac{\partial u}{\partial z} + \frac{\partial v}{\partial z} = 0 \\ \sigma_{xy} = 0 &\Rightarrow \sigma_{xy} = 0 \Rightarrow \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0 \\ \Rightarrow \frac{\partial F_1(y, z)}{\partial y} &= - \frac{\partial F_2(x, z)}{\partial x} \end{aligned}$$

$$\frac{\partial F_1(y, z)}{\partial y} = G_1(z) ; \frac{\partial F_2(x, z)}{\partial x} = G_2(z)$$

$G_1(z) = -G_2(z)$: دالة على z فقط

و لا تنطبق! $\therefore$ $F_1(x, y, z) = G_1(z)y + H_1(z) \quad (4)$

$F_2(x, y, z) = -G_1(z)x + H_2(z) \quad (5)$

من ① و ② و ③ نكتب

$\frac{\partial F_2(x, y, z)}{\partial z} + \frac{\partial F_3(x, y, z)}{\partial y} = 6x \quad (6)$

$\frac{\partial F_1(x, y, z)}{\partial z} + \frac{\partial F_3(x, y, z)}{\partial x} = -6y \quad (7)$

بمقارنة ④ و ⑤ في ⑥ و ⑦ نكتب:

① $-\frac{\partial G_1(z)}{\partial z}x + \frac{\partial H_2(z)}{\partial z} + \frac{\partial F_3(x, y, z)}{\partial y} = 6x$

$\frac{\partial G_1(z)}{\partial z}y + \frac{\partial H_1(z)}{\partial z} + \frac{\partial F_3(x, y, z)}{\partial x} = -6y \quad (8)$

$\begin{cases} -G_1'(z)x + H_2'(z) - 6x = -\frac{\partial F_3(x, y, z)}{\partial y} \\ G_1'(z)y + H_1'(z) + 6y = -\frac{\partial F_3(x, y, z)}{\partial x} \end{cases} \quad (9)$

بما أن $G_1'(z)$ و $H_1'(z)$ و $H_2'(z)$ دالة في z فقط، فإن

$G_1'(z) = A_1 \Rightarrow G_1(z) = A_1 z + B_1$

$H_1'(z) = A_2 \Rightarrow H_1(z) = A_2 z + B_2$

$H_2'(z) = A_3 \Rightarrow H_2(z) = A_3 z + B_3 \quad \text{disg}$

$\begin{cases} -A_1x + A_3 - 6x = -\frac{\partial F_3(x, y, z)}{\partial y} \\ A_1y + A_2 + 6y = -\frac{\partial F_3(x, y, z)}{\partial x} \end{cases}$

$$\begin{cases} -A_1 xy + A_3 y - 2xy = -F_3(x, y) \\ A_1 yx + A_2 x + 6xy = F_3(x, y) \end{cases}$$

$$F_3(x, y) = -\frac{1}{2} (A_2 x + A_3 y) + H_3(y)$$

$$\begin{cases} \frac{\partial F_2(x, z)}{\partial z} - \frac{A_3}{2} = 6x \\ \frac{\partial F_1(y, z)}{\partial z} - \frac{A_2}{2} = -6y \end{cases} \Rightarrow \begin{cases} F_2(x, z) = \frac{1}{2} A_3 z + 6xz + H_3(z) \\ F_1(y, z) = \frac{1}{2} A_2 z - 6yz + H_3(z) \end{cases}$$

$$\varepsilon_x = \frac{\partial u}{\partial x} = 0 \Rightarrow -H_3(z) = 0$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = 0 \Rightarrow H_4(z) = 0$$

$$\begin{aligned} \sigma_{xz} &= -G \sigma_y = G \gamma_{xz} = G \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \sigma_{xz} &= -G \sigma_y = G \gamma_{xz} = G \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \Rightarrow -6y &= \frac{1}{2} A_2 - 6y + \frac{1}{2} A_2 \Rightarrow A_2 = 0 \\ A_3 &= 0 \end{aligned}$$

$$\begin{cases} u = -6yz \\ v = 6xz \\ w = 0 \end{cases}$$

08 03 (w p=0)
 one h x

(1) $w = 0$
 (1) $\frac{\partial w}{\partial x} = 0$

(1) $w = 0$; $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0$
 (1) δ

(1) $M = 0$, $w = 0$
 (1) $\frac{\partial w}{\partial x} = 0$ $V_x = (q_n + \frac{\partial M}{\partial y}) = 0$
 (1)